Total Cost of Preservation (TCP): Cost Modeling for Sustainable Services
Rev. 0.16 – 2012-04-09

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1 Introduction

Information technology and resources are thoroughly integrated with, and indispensable to, today’s Internet and web-based culture, commerce, science, education, and entertainment. The digital assets underpinning these activities, however, are inherently fragile with respect to ever increasing disruptive technological change. Without effective, affordable, and proactive curation management, today’s digital assets will not remain viable and useful in the future. In order to achieve ubiquity of long-term preservation efforts and results, the full economic costs of preservation activities over time, the “total cost of preservation” (TCP), must be well understood.

TCP analysis can be applied usefully in the development of two specific price models for preservation service pricing:

- Pay-as-you-go
- Paid-up

The pay-as-you-go model is appropriate for situations where a reliable and predictable annual income stream is available to the client purchasing preservation services. When this is not the case, for example, with organizations facing irregular or boom-or-bust budgetary cycles or for grant-funded, fixed term research projects, the paid-up model may be more attractive; indeed, in many circumstances it may be the only realistic option.

2 Total cost of preservation

Long-term digital preservation is a complex activity, involving sophisticated technical infrastructure as well as significant human competencies, analysis, and decision making. Given the difficulties in accounting for the myriad aspects of preservation activity, and of projecting the
cost of those activities into the indefinite future, any analysis of TCP must rely on a number of fundamental assumptions.  (See Appendix A for a summary of prior work on preservation cost analysis.)

2.1 Assumptions

The TCP analysis assumes the following:

- Only the costs pertaining to preservation service providers are considered. In terms of the OAIS reference model [OAIS], these are the costs associated with an Archive, “an organization [of people and systems]...that has accepted the responsibility to preserve information.” In particular, the preservation costs associated with the local activities of OAIS Producers (e.g., content creation or acquisition, reformatting, packaging, submission, etc.) are considered out of scope. On the other hand, the cost of supporting a Producer in making use of Archive functions is in scope.

- Individual cost components of the model can be categorized unambiguously as either fixed, which are incurred regardless of level of use, or marginal, which scale proportionally with use.

- Individual cost components can be categorized unambiguously as either one-time or recurring. One-time costs can be annualized over the effective lifespan of the activity or component.

- The values defined for individual cost components represent nominal costs, that is, the costs defined for generic instances of activities or system capacities. This is reasonable under a further assumption of a policy of uniformity of preservation effort rather than outcome. For example, on the basis of form, structure, accompanying metadata, etc., some digital content may be more inherently amenable to preservation care and will naturally receive a higher level of preservation service and outcome.

- Actions performed on preservation content, e.g., characterization, fixity, transformation, etc., are substantially automated. Thus, the main cost associated with an action is in the acquisition – or analysis, design, and development – and deployment of the implementing software, which is independent of the number of objects against which the action is performed.

- The size and scope of the content to be supported is known, and the cost of providing for its preservation is established, at the beginning of the time period under consideration. However, under the pay-as-you-go price model these costs are billed for at the end of the time period while under the paid-up model they are billed for at the beginning of the period.

- The preservation service provider can carry forward budgetary surpluses across fiscal year boundaries. Furthermore, these surplus funds can be invested at market rates, with the
earned interest further contributing to the surplus.

- The models are ultimately revenue neutral, that is, there should be no surplus (or deficit) funds remaining at the end of the period under consideration.

- Values can be determined for various annual adjustment factors, such as inflation, investment rate of return, cost of living adjustments (COLA), merit pay raises, and changes to the per-unit cost of preservation, which, although held constant over the full period of TCP consideration, are nevertheless reliably predictive of long-term economic, technological, and organizational trends.

The last assumption, although the mainstay of the standard economic forecasting technique of discounted cash flow (DCF) analysis [Damodaran], can be problematic over extended time periods. Concerns over the efficacy of DCF-based modeling are addressed in Section § 3.

2.2 TCP analysis

The TCP analysis encompasses the full economic costs associated with the long-term preservation of digital assets, although the resulting price models can be easily customized to deal only with various subsets of those costs as a matter of local policy. The individual cost components incorporated in the analysis are reflective of the following 11 high-level categories abstracted from the environment in which preservation activities occur (see Table 1):

| Preservation activities are embodied in an archival |
|---|---|
| 1. **System**; composed of various |
| 2. **Services** supporting necessary and desirable functions; running on |
| 3. **Servers**; designed, deployed, maintained, enhanced, and utilized by |
| 4. **Staff**; in support of content |
| 5. **Producers**; who use |
| 6. **Workflows** to submit instances of |
| 7. **Content Types**; which occupy |
| 8. **Storage**; and are subject to ongoing |
| 9. **Monitoring**; and periodic |
| 10. **Interventions**; all subject to appropriate managerial |
| 11. **Oversight**. |

Table 1 – Preservation environment

Each of the preceding italicized items is a cost component of the TCP analysis. To simplify that
analysis, the System component encompasses the baseline technical infrastructure and environment in which preservation activities are embodied, and in particular, subsumes the Services that support necessary technical functions and the Servers on which those Services are deployed. Similarly, Staff costs are subsumed by the other individual high-level cost component terms.

The Content Type component encompasses all analysis, planning, and software acquisition or development specific to supporting given class of digital content.

The Monitoring component encompasses technology watch for incipient obsolescence and other risk factors that may impinge on long-term usability; routine pro-active monitoring of the System (and its underlying Services and Servers) and Storage for availability and responsivity, on the other hand, are subsumed under the System and Storage components respectively.

The Intervention component encompasses all non-periodic or unpredictable preservation activities such as format migrations, responses to rights challenges, disaster recovery efforts, etc., storage media refresh and migration are subsumed under the Storage component.

The TCP analysis represents the total cost of preservation to the preservation service provider as follows:

\[
TCP = A + n \cdot P + m \cdot W + \ell \cdot C + k \cdot S + j \cdot M + i \cdot V + O
\]

<table>
<thead>
<tr>
<th>TCP</th>
<th>Total cost of preservation for all Producers.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Fixed cost of the baseline archival System.</td>
</tr>
<tr>
<td>n</td>
<td>Number of content Producers.</td>
</tr>
<tr>
<td>P</td>
<td>Unit cost of supporting a Producer.</td>
</tr>
<tr>
<td>m</td>
<td>Number of submission Workflows.</td>
</tr>
<tr>
<td>W</td>
<td>Unit cost of supporting a Workflow.</td>
</tr>
<tr>
<td>\ell</td>
<td>Number of Content Types.</td>
</tr>
<tr>
<td>C</td>
<td>Unit cost of supporting a Content Type.</td>
</tr>
<tr>
<td>k</td>
<td>Number of units of preservation Storage.</td>
</tr>
<tr>
<td>S</td>
<td>Unit cost of Storage.</td>
</tr>
<tr>
<td>j</td>
<td>Number of preservation Monitoring activities.</td>
</tr>
<tr>
<td>M</td>
<td>Unit cost of a Monitoring activity.</td>
</tr>
<tr>
<td>i</td>
<td>Number of preservation Interventions.</td>
</tr>
<tr>
<td>V</td>
<td>Unit cost of an Intervention.</td>
</tr>
<tr>
<td>O</td>
<td>Fixed cost of administrative and managerial Oversight.</td>
</tr>
</tbody>
</table>

Table 2 – TCP notation
The baseline archival System and administrative/managerial Oversight are considered fixed costs, therefore they are represented in Equation (1) by single terms, A and O, respectively. All other components are marginal costs, each representing by two terms: a unit cost and the number of units allocated or consumed.

The TCP analysis can be used as the basis for partial cost-recovery by removing or zeroing-out particular cost components that are subsidized as a matter of local policy. The specific pay-as-you-go and paid-up price models are based on a derivation of the preservation costs attributable to a given content Producer.

2.3 Pay-as-you-go price model

The pay-as-you-go price model is based on an annual billing cycle, \( t = 0,1,2,\ldots \). Certain cost components – System, Workflow, Content Type, Monitoring, Intervention, and Oversight – are considered “common goods”; that is, they are applicable and beneficial to all Producers equally, and as such, are properly apportioned across all Producers. The number of units of Storage, on the other hand, is specific to a single Producer. The annual pay-as-you-go price for a given Producer is:

\[
G = \frac{A + m \cdot W + \ell \cdot C + j \cdot M + i \cdot V + O}{n} + P + k_P \cdot S
\]

Table 3 – Pay-as-you-go price model notation

The operation of the pay-as-you-go model is illustrated in the cash flow diagram [Neftci] in Figure 1. (Values above the horizontal time axis represent income or price; those below the axis represent expenses or costs.) A series of equal costs \( G \) are incurred at the end of each service year. To achieve revenue neutrality these are matched with corresponding prices.

\[
\begin{array}{cccc}
\text{Income} & G & G & G \\
\hline
\text{Expense} & G & G & G \\
\hline
\end{array}
\]

Figure 1 – Pay-as-you-go price model cash flow diagram

The long-term cost to a Producer under the pay-as-you-go model is found by summing together the annual payments \( G \) over a period of \( T \) years.
Unfortunately, this cumulative price increases linearly as a function of time and, most alarmingly, as $T$ approaches “forever” ($T \rightarrow \infty$), the total price also approaches infinity.

$$G(\infty) = \infty$$

However, recalling the assumption of an annual decrease in the aggregate cost of providing preservation service, $d$, the series of annual costs and prices decreases uniformly, as illustrated in Figure 2.

The size of the annual decrease is proportional to $1-d$, which compounds over time. Again, the long-term cost of is found by summing together the annual payments over $T$ years.

$$G(T, d) = G + G \cdot (1 - d) + G \cdot (1 - d)^2 + \cdots + G \cdot (1 - d)^{T-1}$$

$$G(T, d) = G \cdot \frac{1 - (1 - d)^T}{d}$$

Mathematically, Equations (7) and (8) are the solutions of finite and infinite power series.
respectively, which are solvable only when the rate of decrease in aggregate preservation service cost, $1-d$, is less than 1. For details of the derivation of these equations, see Appendices B and C. For the derivation of the value of $d$ see Appendix D.

2.3  Paid-up price model

The paid-up price model is based on a one-time price, paid at the beginning of the period under consideration, that is sufficient to finance all subsequent preservation activities over that period. This period can be either a fixed term $T$ or “forever,” that is, for $T = \infty$. In the relevant cash flow diagram in Figure 3, the one-time payment $F$ is paid by the Producer at the beginning of year 1 ($t = 0$). The subsequent income terms are the result of the investment return $r$ applied to each year’s surplus funds. Note that $r$ is the nominal annual percentage rate (APR) of investment return. The effective annual rate, $e$, incorporating the effect of monthly compounding, is [Tucker]:

$$e = \left(1 + \frac{r}{12}\right)^{12} - 1$$

(9)

The pay-as-you-go cost $G$ is scaled by a factor of $e/r$ to account for the fact that the service provider incurs costs throughout the service year but these are not billed and paid for until the end of the year.

<table>
<thead>
<tr>
<th>Income</th>
<th>$F$</th>
<th>$r \cdot F$</th>
<th>$r \cdot [(1 + r) \cdot F - G] - (1 - d) \cdot G$</th>
<th>$r \cdot [(1 + r) \cdot [(1 + r) \cdot F - G] - (1 - d) \cdot G - (1 - d) \cdot G]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Expense</td>
<td>$G$</td>
<td>$(1 - d) \cdot G$</td>
<td>$(1 - d)^2 \cdot G$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3 – Paid-up price model cash flow diagram

The paid-up price $F$ is derived in Appendix E.

$$F(T, d, r) = G \cdot \frac{e}{r} \cdot \frac{1}{1 + e} + G \cdot \frac{e}{r} \cdot \frac{(1 - d)}{(1 + e)^2} + G \cdot \frac{e}{r} \cdot \frac{(1 - d)^2}{(1 + e)^3} + \cdots + G \cdot \frac{e}{r} \cdot \frac{(1 - d)^{T-1}}{(1 + e)^T}$$

(10)

$$F(T, d, r) = G \cdot \frac{e}{r} \cdot \frac{(1 + e)^T - (1 - d)^T}{(1 + e)^T \cdot (e + d)}$$

(11)

$$F(\infty, d, r) = G \cdot \frac{e}{r} \cdot \frac{1}{e + d}$$

(12)
\[ F(T,d,r) \quad \text{Paid-up price for } T \text{ years.} \]

- \( r \): The nominal annual percentage rate (APR) of investment return (as a decimal, for example, 0.02 for 2%).
- \( e \): The effective annual percentage rate of investment return (as a decimal).

\[ \phi = \frac{e}{r \cdot (e + d)} \]

Table 6 – Paid-up price model notation

Equations (11) and (12) are also power series and are solvable only when the discount factor is less than the effective investment return, \( 1 - d < 1 + e \). For details of the derivation of these equations, see Appendix E.

It is desirable for a Producer to be able to switch easily between a pay-as-you-go and paid-up basis. The relevant factor in this transition is the “coefficient of permanence,” that is, the one-time premium, or multiplier, of the annual pay-as-you-go price, \( G \), that one must incur in order to achieve paid-up permanence.

\[ \phi \quad \text{Coefficient of permanence; the one-time multiplier of the pay-as-you-go price necessary to achieve paid-up permanence.} \]

Table 7 – Paid-up model notation (continued)

Thus, a Producer currently paying \( G \) on an annual basis can “trade up” to service permanence with a one-time payment of \( F = \frac{G \cdot e}{r \cdot (e + d)} \).

The paid-up model is designed to be ultimately revenue neutral, that is, with no surplus (or deficit) funds remaining at the end of the term. This can be illustrated in the following example for a ten year term \( (T = 10) \), a pay-as-you-go price \( G \) of $650 (for 1 TB of storage), a nominal annual investment rate of return of 2% \( (r = 0.02) \), and an annual decrease in the aggregate cost of preservation service of 10% \( (d = 0.1) \). From Equation (11), this leads to a paid-up price \( F \) of $4,764.14. Income in years 1, 2, 3,... is the result of the investment return on the surplus remaining at the end of the prior year.

<table>
<thead>
<tr>
<th>Year ((T))</th>
<th>Income</th>
<th>Expense</th>
<th>Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4,764.14</td>
<td>$0.00</td>
<td>$4,764.14</td>
</tr>
<tr>
<td>1</td>
<td>$96.16</td>
<td>$655.99</td>
<td>$4,204.31</td>
</tr>
<tr>
<td>2</td>
<td>$84.86</td>
<td>$623.19</td>
<td>$3,665.98</td>
</tr>
<tr>
<td>3</td>
<td>$73.00</td>
<td>$592.03</td>
<td>$3,147.94</td>
</tr>
</tbody>
</table>
The relationship between the pay-as-you-go and paid-up models is illustrated in Figure 4. The cumulative and discounted pay-as-you-go prices and the paid-up price, \( G(T), G(T,d), F(T,d,r) \), are shown as the red, orange, and green curves, respectively. The cumulative pay-as-you-go and paid-up prices for “forever,” \( G(\infty,d) \) and \( F(\infty,d,r) \), are shown in magenta and blue, respectively.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cumulative Pay-as-you-go Price, ( G(T) )</th>
<th>Discounted Pay-as-you-go Price, ( G(T,d) )</th>
<th>Discounted Pay-as-you-go Price, ( G(\infty,d) )</th>
<th>Paid-up Price, ( F(T,d,r) )</th>
<th>Paid-up Price, ( F(\infty,d,r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$63.54</td>
<td>$562.43</td>
<td>$2,649.05</td>
<td>$2,649.05</td>
<td>$2,649.05</td>
</tr>
<tr>
<td>5</td>
<td>$53.47</td>
<td>$534.31</td>
<td>$2,168.21</td>
<td>$2,168.21</td>
<td>$2,168.21</td>
</tr>
<tr>
<td>6</td>
<td>$43.76</td>
<td>$507.59</td>
<td>$1,704.38</td>
<td>$1,704.38</td>
<td>$1,704.38</td>
</tr>
<tr>
<td>7</td>
<td>$34.40</td>
<td>$482.21</td>
<td>$1,256.57</td>
<td>$1,256.57</td>
<td>$1,256.57</td>
</tr>
<tr>
<td>8</td>
<td>$25.36</td>
<td>$458.10</td>
<td>$823.83</td>
<td>$823.83</td>
<td>$823.83</td>
</tr>
<tr>
<td>9</td>
<td>$16.63</td>
<td>$435.20</td>
<td>$405.26</td>
<td>$405.26</td>
<td>$405.26</td>
</tr>
<tr>
<td>10</td>
<td>$8.18</td>
<td>$413.44</td>
<td>$0.00</td>
<td>$0.00</td>
<td>$0.00</td>
</tr>
</tbody>
</table>

Table 8 – 10 year term paid-up price model example

The difference between the cumulative (red) and discounted (orange) pay-as-you-go prices is accounted for by the discounting factor \( d \), the assumed annual decrease in the cost of providing preservation service. The difference between the fixed term pay-as-you-go (orange) and paid-up (green) prices, as well as the “forever” prices (magenta and blue), is accounted for by \( r \); the assumed annual investment rate of return.

The curve for the fixed term paid-up price characteristically approaches the forever price asymptotically so long as the investment rate of return and annual decrease in aggregate...
preservation service cost are greater than 0 ($r > 0$ and $d > 0$), or if the aggregate cost increases (i.e., $d < 0$), when the investment gain is greater than the service cost increase ($r > |d|$).

### 2.4 Rate of change of preservation service cost

The variable $d$ in Equations (7), (8), (11), and (12) is a composite value or function that sums the individual component-specific changes in quantity and unit cost, weighted by the component’s proportionate contribution to the overall cost (see Appendix E).

### 3 Predicative reliability

Many of the assumptions built into the TCP model may be problematic over significant time spans, particularly the assumption that the various annual adjustment factors – investment rate, rate of decrease of unit cost, inflation, COLA, etc. – are constant over time [Rosenthal].

Several avenues are available to ameliorate these difficulties.

#### 3.1 Model the risk

The values for $r$ and $d$ can be rounded up (or down) as appropriate to represent a fixed risk “premium” for the future uncertainty of financial and technological trends. A better approach is to add an additional risk component $R$ to the pay-as-you-go Equation (2).

$$ G = \frac{A + m \cdot W + \ell \cdot C + j \cdot M + i \cdot V + O + R}{n} + P + k_p \cdot S \quad (14) $$

| $R$ | Risk cost posed by uncertainty over the future values of $r$ and $d$. |

The growing influence of $R$ over time, reflective of increasing uncertainty, can be controlled by setting the appropriate value for $\omega_R$ and $d_R$, the weighting factor and expected annual rate of change in risk used in Equation (14) for $d$, the rate of change for the aggregate cost of preservation in Equations (7), (8), (11), and (12).

Insurers protect themselves from truly unexpectedly large payouts by purchasing reinsurance: in essence, insurance against excessive claims [Reinsurance]. The additional cost introduced into the pricing equations through the risk factor $R$ can be seen as representing a reinsurance “premium” that accumulates over time in anticipation of meeting some future unexpected need. This is also similar to the idea of a “rainy day fund” that governments fund during periods of budgetary surplus in anticipation of future shortfalls [Rainy].
3.2  Recalibration

Rather than keeping key model variables, \( F, G, r, \) and \( d \), constant over time, they can be recalibrated periodically on the basis of cumulative experience and current knowledge. Recalibrated prices would apply for prospective Producers only; retrospective Producers would remain “locked in” at the price in force at the time they established their relationship with the service provider.

3.3  Hybrid pricing

The hybrid approach to pricing distinguishes between those cost components that are considered easy to quantify and forecast, and those that aren’t. Paid-up pricing is applied only to the former; activities associated with the latter are performed, if it all, on a pay-as-you-go basis.

For example preservation Interventions, future actions taken in response to future, possibly unanticipated, conditions, may be considered to fall into the difficult-to-quantify category. The paid-up price would therefore only incorporate the costs for preservation activities that can be performed at the time of content ingest via Workflows as the result of Content Type analysis, such as the creation of normalized or desiccated [WAS, Desiccation] derivative forms of the contributed content.

In a more extreme case, the baseline paid-up preservation function could be restricted to bit-level preservation only. This service mode requires only three cost components: the System, Storage, and Producer; support for Workflow is not necessary as the single required workflow is assumed to be subsumed under the System, while Content Types, Monitoring, and Interventions are not necessary since only bit-level activities are encompassed at this service level.

3.4  Shorten the term

Uncertainty in the predictive reliability of the values chosen for \( r \) and \( d \) is proportion to the length of time under consideration; the longer the term, the greater the uncertainty. With this in mind, it may be prudent to shorten the term to one more amenable to reliable forecasting, with the possibility for subsequent “renewal” of preservation service. For some bodies of content a fixed term – 10 year? 20 years? – may in fact be an appropriate initial term. Short to mid-term preservation provides an opportunity for the content to prove its value, as evidenced by the commitment of a curatorial advocate to pay for its subsequent preservation.

3.5  Stochastic modeling

Rosenthal suggests using more sophisticated economic techniques based on Monte Carlo simulation to model potential future scenarios stochastically [Rosenthal].
Appendix A  Prior work

A number of international efforts have studied the question of long-term preservation costs; most notably, the Nationaal Archief of the Netherlands in 2005 [Archief]; the LIFE² project (Life Cycle Information for E-Literature) work on a Generic Preservation Model (GPM) in 2008 [Ayris, Hole, LIFE]; the KRDS project (Keeping Research Data Safe) in 2010 [Beagrie, KRDS]; the Princeton DataSpace initiative in 2010 [Goldstein]; and the Danish National Archives and Royal Library CMDP (Cost Model for Digital Preservation) in 2011 [Kejser, CMDP]. (See [Zeller] for a summary of these and other relevant activities.)

All of these models analyze preservation costs throughout the full lifecycle of preserved assets. Both the Nationaal Archief and LIFE project work employ very fine-grained analysis of cost components and are based on representative actual costs rather than the nominal costs employed by TCP. The LIFE model includes cost components for content creation and acquisition, which are considered out of scope in the TCP analysis. Furthermore, LIFE scales preservation action costs by the number of expected objects. TCP assumes that there is no per-object marginal cost; instead, marginal costs are associated only with the various Content Types of which objects are members. The KRDS model is specific to the research data lifecycle, but its findings appear to be applicable to other contexts. Unlike the the Nationaal Archief and LIFE models, KRDS assumes a discounting function that annually decreases the aggregate cost of preservation service. The DataSpace model is also based on a discounting function, but its analysis covers only the costs associated with preservation storage, which are defined on a pay once, store forever (POSF) basis. The OAIS activity-based CMDP work concentrates on post facto measuring of preservation costs, rather than on forecasting, although it is possible using the framework. While DataSpace is explicitly concerned with “forever” pricing, neither it nor any of the other models assume the benefit of an annual investment return in offsetting a portion of ongoing costs. Most, if not all, of the individual cost components of these models can be mapped to OAIS environmental and functional entities [OAIS] or TRAC criteria [TRAC], facilitating common points of reference and comparison.

A number of institutions, both commercial and non-profit/academic, now offer long-term preservation services. (Or at least persistent storage; the important distinction between the two is not always apparent from the description of these service offerings.) Carbonite is representative of commercial preservation service offerings [Carbonite]. Its cost is $599/year for up to 500 GB, with further increments of 100 GB available for $89/year. The LifeTime Library at the University of North Carolina offers students permanent storage “and associated services” for 250 GB, apparently with no associated fees [UNC]. The USC Digital Repository offers a paid up license for 20 years of preservation service for $1000/TB [USC]. In general, however, little information is available explaining the basis of their business models.
Appendix B  Power series

The solution for a general power series, \( p(x) = \sum_{i=0}^{T-1} a \cdot x^i \), can be derived as follows:

\[
p(x) = a + a \cdot x + a \cdot x^2 + \cdots + a \cdot x^{T-1}
\]  \( \text{(15)} \)

Equation (15) can be scaled by a factor of \( x \):

\[
x \cdot p(x) = a \cdot x + a \cdot x^2 + \cdots + a \cdot x^{T-1} + a \cdot x^T
\]  \( \text{(16)} \)

Subtracting Equation (16) from (15) yields:

\[
p(x) - x \cdot p(x) = a + (a \cdot x - a \cdot x) + \cdots + (a \cdot x^{T-1} - a \cdot x^{T-1}) - a \cdot x^T
\]

\[
p(x) - x \cdot p(x) = a - a \cdot x^T
\]  \( \text{(18)} \)

\[
p(x) \cdot (1 - x) = a \cdot (1 - x^T)
\]  \( \text{(19)} \)

Thus, dividing Equation (19) by \( 1-x \):

\[
p(x) = a \cdot \frac{1 - x^T}{1 - x} \quad \text{where} \quad x \neq 1
\]  \( \text{(20)} \)

The equation for \( p(x) \) as an infinite series is derived by observing that \( x^T \) in Equation (20) approaches 0 as \( T \) approaches \( \infty \), so long as \( 0 \leq x < 1 \).

\[
\lim_{T \to \infty} p(x) = \frac{a - 0}{1 - x}
\]  \( \text{(21)} \)

\[
\lim_{T \to \infty} p(x) = \frac{a}{1 - x} \quad \text{where} \quad 0 \leq x < 1
\]  \( \text{(22)} \)

Various other equations in the TCP models are derived by substituting the appropriate terms for \( a \) and \( x \) in Equations (20) and (22).
Appendix C Pay-as-you-go price model

The cumulative discounted pay-as-you-go price over $T$ years is based on the sum of a geometrically decreasing series of payments, each proportionally discounted from $G$ by a factor of $1-d$ (see Figure 2):

$$G(T, d) = G + G \cdot (1 - d) + G \cdot (1 - d)^2 + \cdots + G \cdot (1 - d)^{T-1}$$  \hspace{1cm} (23)

The solution for Equation (23) is simply the general solution for a finite power series given by Equation (20) with $G$ substituted for $a$ and $1-d$ for $x$.

$$G(T, d) = a \cdot \frac{1 - x^T}{1 - x} \quad \text{where} \quad a = G \quad \text{and} \quad x = 1 - d$$  \hspace{1cm} (24)

Substituting for $a$ and $x$ and simplifying produces the cumulative discounted pay-as-you-go price over $T$ years.

$$G(T, d) = G \cdot \frac{1 - (1 - d)^T}{d} \quad \text{where} \quad d \neq 0$$  \hspace{1cm} (25)

Likewise, the finite power series also has a solution for the case where time is arbitrarily large as long as it satisfies the convergence criterion, $0 \leq x < 1$.

$$G(\infty, d) = \frac{a}{1 - x} \quad \text{where} \quad a = G \quad \text{and} \quad x = 1 - d$$  \hspace{1cm} (26)

Substituting in for $a$ and $x$ and simplifying produces the cumulative discounted pay-as-you-go price for “forever.”

$$G(\infty, d) = \frac{G}{d} \quad \text{where} \quad 0 \leq 1 - d < 1$$  \hspace{1cm} (27)
Appendix D  Discount factor

The variable \( d \) in Equations (7), (8), (11), and (12) is a composite value or function that sums the individual component-specific changes in quantity and unit cost, weighted by the component’s proportionate contribution to the overall cost (see Appendix E).

\[
d = \omega_A \cdot d_A + \omega_W \cdot (d_m \cdot d_W) + \omega_C \cdot (d_C \cdot d_C) + \omega_M \cdot (d_M \cdot d_M) + \omega_V \cdot (d_I \cdot d_V) + \omega_S \cdot (d_S \cdot d_S) + \omega_O \cdot d_O
\]  

where

\[
\omega_A = \frac{A}{n \cdot G}
\]  
\[
\omega_W = \frac{m \cdot W}{n \cdot G}
\]  
\[
\omega_C = \frac{\ell \cdot C}{n \cdot G}
\]  
\[
\omega_M = \frac{j \cdot M}{n \cdot G}
\]  
\[
\omega_V = \frac{i \cdot V}{n \cdot G}
\]  
\[
\omega_S = \frac{k \cdot S}{G}
\]  
\[
\omega_O = \frac{A}{n \cdot G}
\]

| \( d \) | Annual percentage rate of change in aggregate preservation service cost. |
| \( \omega_A \) | Weighting factor for the System component. |
| \( d_A \) | Annual percentage rate of change in fixed cost of the archival System. |
| \( \omega_W \) | Weighting factor for the Workflow component. |
| \( d_m \) | Annual percentage rate of change in the number of supported Workflows. |
| \( \omega_C \) | Weighting factor for the Content Type component. |
| \( d_C \) | Annual percentage rate of change in the nominal unit cost of a Workflow. |
| \( \omega_M \) | Weighting factor for the Monitoring component. |
| \( d_M \) | Annual percentage rate of change in the number of supported Content Types. |
| \( \omega_V \) | Weighting factor for the Intervention component. |
| \( d_C \) | Annual percentage rate of change in the number of supported Content Types. |
| \( \omega_M \) | Weighting factor for the Monitoring component. |
\[
\begin{align*}
    d_i & \quad \text{Annual percentage rate of change in the number of supported } \text{Interventions}. \\
    d_v & \quad \text{Annual percentage rate of change in the nominal unit cost of an } \text{Intervention}. \\
    \omega_S & \quad \text{Weighting factor for the } \text{Storage} \text{ component}. \\
    d_k & \quad \text{Annual percentage rate of change in the number of } \text{Storage} \text{ units consumed}. \\
    d_S & \quad \text{Annual percentage rate of change in the nominal cost of a unit of } \text{Storage}. \\
    \omega_O & \quad \text{Weighting factor for the } \text{Oversight} \text{ component}. \\
    d_O & \quad \text{Annual percentage rate of change in the number cost of a unit of } \text{Oversight}.
\end{align*}
\]

Table 11 – Change in preservation service cost notation

Since the System and Oversight are fixed costs, the numerators of Equations (29) and (35) only have the unit cost term; all other weighting factors have both number of units and unit cost terms. Since the costs for System, Workflows, Content Types, Monitoring, Interventions, and Oversight are common goods, they are apportioned across all \(n\) Producers, as reflected in the denominators of Equations (29) – (33) and (35); Storage is always specific to a given Producer, so there is no \(n\) term in Equation (34).

Note that all of the discount factors are real rates of change, incorporating the effects, if any, of inflation.
Appendix E  Paid-up price model

Like the pay-as-you-go price, the paid-up price over $T$ years is based on the sum of a geometrically decreasing series of payments (see Figure 3). The present value $F$ of the future series of payments using the base cost $G$ declining annually by $1-d$, and a nominal investment return $r$ compounded monthly is:

$$F(T, d, r) = \frac{G \cdot e}{1 + e} \cdot \frac{1}{r} \left( 1 + \frac{1 - d}{1 + e} \right) + \frac{G \cdot e}{r} \cdot \frac{(1 - d)^2}{(1 + e)^2} + ... + \frac{G \cdot e}{r} \cdot \frac{(1 - d)^{T-1}}{(1 + e)^{T-1}}$$

(36)

The effective investment return $e$, incorporating the effect of monthly compounding, is [Tucker]:

$$1 + e = \left( 1 + \frac{r}{12} \right)^{12}$$

(37)

Each declining term, $G \cdot (1 - d)^t$, representing a future payment is converted to its present value by adjusting for the monthly-compounded time value of money, that is, scaling by $\frac{e}{r} \cdot (1 + e)^{t+1}$, and the results are summed to produce the total present value. Rearranging the terms of Equation (36) gives:

$$F(T, d, r) = \frac{G}{1 + e} \cdot \frac{e}{r} \left( 1 + \frac{1 - d}{1 + e} + \frac{(1 - d)^2}{(1 + e)^2} + ... + \frac{(1 - d)^{T-1}}{(1 + e)^{T-1}} \right)$$

(38)

The solution to Equation (38), is simply the general solution to the power series given by Equation (20) where $a$ is $\frac{G}{1 + e} \cdot \frac{e}{r}$ and $x$ is $\frac{1 - d}{1 + e}$

$$F(T, d, r) = a \cdot \frac{1 - x^T}{1 - x} \text{ where } a = \frac{G}{1 + e} \cdot \frac{e}{r} \text{ and } x = \frac{1 - d}{1 + e}$$

(39)

Substituting for $a$ and $x$ and simplifying produces the paid-up price for $T$ years.

$$F(T, d, r) = \frac{G \cdot e}{r} \cdot \frac{(1 + e)^T - (1 - d)^T}{(1 + e)^T \cdot (e + d)} \text{ } \Box$$

(40)

Likewise, the finite power series also has a solution for the case where time is large as long as it satisfies the convergence criterion, $0 \leq x < 1$.

$$F(\infty, d, r) = \frac{a}{1 - x} \text{ where } a = \frac{G}{1 + e} \cdot \frac{e}{r} \text{ and } x = \frac{1 - d}{1 + e}$$

(41)

Substituting for $a$ and $x$ and simplifying produces the paid-up price for “forever.”
\[ F(\infty, d, r) = \frac{e}{r} \cdot \frac{G}{e + d} \text{ where } 0 \leq \frac{1 - d}{1 + e} < 1 \]
References


